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Efficient Parametrization of Generic Aircraft Geometry

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A new method is presented for the parametrization of aircraft geometry; it is efficient in the sense that a relatively small number of "design" parameters are required to describe a complex surface geometry. The method views surface generation as a boundary-value problem and produces surfaces as the solutions to elliptic partial differential equations, hence its name, the PDE method. The use of the PDE method will be illustrated in this article by the parametrization of double delta geometries; it will be shown that it is possible to capture the basic features of the large-scale geometry of the aircraft in terms of a small set of design variables.

Nomenclature

a	= smoothing parameter
a_{xi}	= $\partial a/\partial xi$
$a(\chi)$	= thickness of fuselage at parametric position χ
$a\omega$	= parameter determining degree of "washout" in wings
a_0, a_1	= constants determining shape of fuselage
(b/ch)	= scaling between character lines 1 and 2
ch	= parameter setting chord length of airfoil
$D^m_{u,v}(X)$	= partial differential operator of order m
$F(u, v)$	= vector valued function
h_1	= parameter setting span of inner portion of wing
h_2	= parameter setting span of outer portion of wing
rl	= scaling constant
swt	= parameter controlling swallow-tail effect
sxt	= parameter controlling magnitude of surface derivatives at wing
$s1$	= adjustable design parameter
$s2$	= adjustable design parameter
t	= parameter setting thickness of airfoil
u	= parametric surface coordinate
v	= parametric surface coordinate
X_u	= $\partial X/\partial u$
$X(u, v)$	= parametrically defined surface patch in physical space
$(-x_d, y_d)$	= offset between character lines 1 and 2
$(y_f)_\theta$	= $\partial y_f/\partial\theta$
$[x_f(\theta), y_f(\theta), z_f(\theta)]$	= parametric curve at junction of fuselage and inner wings
xt	= parameter setting fore/aft position of wingtip
xte	= position of fuselage with respect to character lines 1 and 2
xtl	= parameter setting length of wingtip
$[x_t(\theta), y_t(\theta), z_t(\theta)]$	= parametric curve at tip of outer wing
$[x(\theta), y(\theta), z(\theta)]$	= parametric curve at junction of outer and inner wings
$\partial\Omega$	= boundary of Ω
χ	= parameter setting position along fuselage
Ω	= finite domain in (u, v) parameter space

I. Introduction

ACCORDING to Sloof,[1] traditional aerodynamic design can be broken down into two phases: 1) preliminary design, in which the major design variables such as general dimensions, wing loading, basic wing planform, etc., are chosen and 2) detail design, in which the detailed geometry of the wings and other components of the airframe are decided upon. On the other hand, Raymer,[2] discussing wider aspects of aircraft design, distinguishes three phases in the design process: 1) conceptual design, in which basic questions concerning configuration, size, weight, and performance are answered; 2) preliminary design, in which the configuration becomes fixed, quantitative geometric design starts, and major items such as landing gear, propulsion, and control systems are designed; and 3) detail design, during which the actual pieces from which the aircraft will be built are designed and the question of how the aircraft will be put together is considered.

Despite the differences in these two descriptions of aircraft design, both identify an early stage in the design process during which general questions concerning the aircraft's configuration are considered; when, in order to meet requirements, various alternative design solutions must be considered. In the past, when considering the question of the physical properties of new designs, designers have had to rely upon their own knowledge and experience, and, further along in the design process, model testing. However, the increasing sophistication of numerical methods and the increasing power of computer hardware have meant that the properties of new designs can be analyzed by computer long before any physical embodiment is created.[3,4] Furthermore, whereas the main use of numerical methods has been as an alternative to model testing, there is an increasing trend towards their use in the design process as a tool for optimization.[4–6]

As far as numerical techniques for aerodynamic design are concerned, Labrujere and Sloof[4] distinguish two classes of approach: 1) inverse design and 2) direct numerical optimization. The object of inverse design is to take a given pressure distribution and then solve for a geometry that produces that distribution. Direct numerical optimization, on the other hand, involves coupling an aerodynamic analysis method with a

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scheme for numerical minimization. The analysis method is used to calculate the value of a function that, in some sense, characterizes the aerodynamics of the aircraft. This "measure-of-merit" or "objective" function can be regarded as a function of whatever variables have been used to parametrize the aircraft's geometry, and the task of the minimization method is to find an optimum set of shape parameters that minimizes this objective function. Thus, starting from some initial design, the combination of computer codes for analysis and optimization iteratively moves through the parameter space defined by the shape variables, each time producing a design with an improved performance, eventually resulting in a design that meets requirements.

As various authors have pointed out,[1,3,4] the great advantages of direct numerical optimization are its flexibility when it comes to allowing for the existence of constraints on the design, its flexibility with respect to the selection of design objectives, and the capabilities it offers for multipoint design. However, the major disadvantage of the method is the great computational cost of each iterative step, which is incurred by the fact that the physical calculations must be carried out accurately and which grows as the number of design variables increases. Thus, given the present capabilities of computer hardware, it is of paramount importance to limit the number of variables characterizing a design. For this reason, Labrujere and Sloof[4] report that although direct numerical optimization of such global objective functions as lift-to-drag ratio is feasible in two-dimensional problems, in three dimensions the computational cost of such an approach is still daunting. In fact, according to these authors, in "3-D wing design especially, the number of design variables is so large that the practical application of the concept seems to be remote. . . ."

Despite its computational costs, a number of authors have considered direct numerical optimization by trying to limit the number of design variables. Cosentino and Holt,[7] for example, consider the optimized design of transonic wing configurations by representing the two-dimensional airfoils that define a wing geometry in terms of spline curves, and then using the position of the spline control points, in particular those points which affect the wing region wetted by supersonic flow, as design variables to be optimized. Destarac and Reneaux,[8] considering similar problems, adopted a number of alternative approaches. In the case of the design by numerical optimization of a supercritical airfoil, they produced optimized airfoil shapes as the linear combination of existing supercritical airfoils from a library, the design variables in this case being the "scaling factor" for each component airfoil. In the same paper, they also describe the reduction of wing/engine interference where the design variables used were "aerofunctions": shapes whose effect on the pressure distribution around a two-dimensional airfoil was known from inverse methods. This latter approach, the use as design variables of perturbation shapes having a known effect on the pressure distribution around a baseline shape, was developed by Aidala et al.[9] One of the advantages that they claimed for the approach was the fact that such design variables have a direct physical significance for the designer, thus helping him/her to make a more effective choice of design parameters, and hence, limiting the computational cost.

However, before the computer-aided analysis and optimization of the physical properties of a new design can begin, the geometry of the initial concept must be converted to a representation in a computer, which nowadays, according to Raymer,[2] often starts with a design drawing carried out with a computer-aided drafting system. A design drawing describes the aircraft's geometry in terms of "character lines" on its surface. Obviously, these constitute only a partial definition of the aircraft's surface, and an unambiguous geometric representation only comes after the process of "lofting," effectively the creation of surfaces between the character lines, has been carried out. Thus, there is a need for mathematical methods for representing or parametrizing curves and surfaces, which are flexible enough to represent a wide range of shapes in an easy and intuitive manner. If a new design is close to an existing design, then the old design can usefully be used as the starting point for the description of the new, since any modifications are likely to be relatively minor. However, when a completely new design is needed and the designer must investigate a wide range of radical new shapes, then a method for shape parametrization is required with enough scope to enable the designer to choose from a wide spectrum of different design possibilities.

We have discussed a number of methods that have been used to parametrize geometry for the purposes of numerical optimization. These methods implicitly assume that the new optimized design will be close to the initial design point, and furthermore they are concerned not with the global parametrization of the aircraft's surface, but with the parametrization of a (fairly) localized part of its geometry. When seeking to make improvements to an existing design, such an assumption is justified, but when conducting a design study of a radically new concept, especially for the whole aircraft, as we have noted earlier, there is a need for quickly and cheaply defining a wide range of new shapes. This is especially true at present, given the increasing tendency to use numerical optimization, with a consequent need to limit the number of design variables, at an earlier and earlier stage in the design process.

The rest of this article presents a new method for the efficient parametrization of complex geometries, efficient in the sense that it can present complex three-dimensional surfaces in terms of a relatively small set of design variables. The method is known as the PDE method because of its use of elliptic partial differential equations to generate surfaces. It was developed in the area of computer aided geometric design[10,11] and has been used as the basis of automatic design for function, i.e., numerical optimization, in a variety of contexts.[12-16]

In this article we shall confine ourselves to describing how the method may be used to parametrize geometry, using as our example the sort of double delta configuration that is being considered for supersonic transports.[5]

II. PDE Method

The PDE method was introduced into the area of computer-aided geometric design as a method for blend generation.[10] The problem of blend generation is essentially that of being able to generate a smooth surface to act as a bridging transition between neighboring "primary" surfaces. This may be necessary in order to satisfy aesthetic considerations, or for more functional reasons such as the need to relieve stress concentrations or so that the object can be machined. Four major categories of blends have been distinguished: blending surfaces governed by strong functional constraints, aesthetic blends, fairings, rounds, and fillets, and there are a number of methods currently available for producing blend surfaces.[17]

Mathematically, the calculation of a blending surface may be formulated as a classic boundary-value problem, in that we require a function X defined over a domain Ω in two-dimensional parameter space that satisfies specified boundary data around the edge region $\partial\Omega$. In the case of blending, this boundary data will typically be in the form of X and a number of its derivatives specified on $\partial\Omega$. The order of the PDE, and hence, the number of derivatives specified, will be determined by the required degree of continuity between the blend and the surfaces to which it joins. In the PDE approach we view u and v as coordinates of a point in Ω and the function X as a mapping from that point in Ω to a point in physical space. To satisfy these requirements we regard X as the solution of a PDE

$$D^{m}_{u,v}(X) = F(u, v) \qquad (1)$$

where $D^m_{u,v}(X)$ is a partial differential operator of order m in the independent variables u and v, while $\boldsymbol{F}$ is a vector valued function of u and v. Since we are considering boundary-value problems it is natural to consider the class of elliptic PDEs.

Past work has concentrated upon solutions to the following equation:

$$\left(\frac{\partial^2}{\partial u^2} + a^2 \frac{\partial^2}{\partial v^2}\right)^2 X = 0 \qquad (2)$$

This equation is solved over some finite region Ω of the (u, v) parameter plane, subject to boundary conditions on the solution that usually specify how X and its normal derivative $\partial X/\partial n$ vary along $\partial\Omega$. The three components of the function X $[(x(u, v), y(u, v), z(u, v)]$ are the Euclidean coordinate functions of points on the surface given parametrically in terms of the two parameters u and v, which define a coordinate system on the surface. Note that in the simplest cases Eq. (2) is solved independently for the x, y, and z coordinates.

The boundary conditions on X, which we shall refer to as function boundary conditions, determine the shape of the curves bounding the surface patch in physical space, or more specifically, their parametrization in terms of u and v. The boundary conditions on $\partial X/\partial n$, which we shall refer to as derivative boundary conditions, basically determine the direction in which the surface moves away from a boundary and how "fast" it does so.

The partial differential operator in Eq. (2) represents a smoothing process in which the value of the function at any point on the surface is, in a certain sense, an average of the surrounding values. In this way a surface is obtained as a smooth transition between the boundary conditions imposed on the function and its first derivative. The parameter a controls the relative rates of smoothing between the u and v parameter directions, and for this reason has been called the smoothing parameter in earlier work.[18]

The original work on blending was extended to cover free-form surface design[11] and it was demonstrated that the method is capable of producing surfaces of practical significance such as propellers and ship hulls.[19–21]

The major difficulty in the analysis inherent in the design of engineering surfaces is being able to represent them by a method that involves few design parameters and which allows easy manipulation in a predictable way.[22] The PDE method is not only capable of producing functionally useful surfaces, but also of parametrizing them in terms of a relatively small number of design parameters. The reason for this is that it adopts a boundary-value approach to surface design, and therefore, surfaces are defined by data distributed around their edges rather than across their entire surface area. Being so few in number it is a practicable proposition to optimize computationally the surface with respect to its intended functionality. Lowe et al.[12] have shown how this can be achieved by minimization techniques in the phase space defined by the parameters describing the shape of the object. The problems they considered were that of heat transfer, strength, and hydrodynamic performance (see also Refs. 13–16). The aim of this research is to develop a methodology whereby an initial design parametrized by the PDE method can automatically be optimized against some suitably defined functional criterion, subject to constraints prespecified by the designer in order to ensure a sensible final design. This work is an attempt to meet the need expressed by Shapiro and Voelcker[23] for a systematic way of considering the relationship between geometry and function.

Even though the PDE method is unlike other methods of surface design, work has been carried out that indicates how it can be integrated with conventional surface modeling systems. In particular, earlier work has shown how PDE surfaces may be calculated in terms of conventional (B-)spline surfaces by using collocation and finite element techniques.[24,25] In this way, one can calculate surfaces having the desirable properties of PDE surfaces (very smooth complex surfaces parameterized by few design variables) and the desirable properties of B-spline surfaces (e.g., local control over limited regions of the surface allowing for any "fine-tuning" that might be necessary in the surface shape).

Although the PDE method was originally envisaged as a technique for surface generation rather than representation, work has been carried out using the PDE method to create a surface model of an actual marine propeller. Initially, the method was used to create a qualitative representation of the surface, and then extended to a quantitative fit with data describing a real propeller.[20,21] According to current practice, propeller blades are usually defined by specified airfoil sections placed at certain stations along the span of the blade. These cross sections are arranged so as to produce the desired load distribution for an efficient propeller. In contrast to this, the PDE method produces a marine propeller blade by a boundary-value approach where the two boundaries between which the surface is formed are a curve with an airfoil shape at the root of the blade (near where it joins to the boss), and a curve of vanishingly small size at the tip of the blade. By a suitable choice of derivative boundary conditions a shape resembling a marine propeller can be obtained.

III. Definition of a Generic Aircraft

As outlined in Sec. II, the PDE method has been used in a number of different application areas, e.g., the design of ship hulls and marine propellers.[13,14,20] In this earlier work a gradual approach was adopted. The method was introduced by showing how it could define generic geometries by using simplified boundary conditions.[11,19] It was then shown how it could produce geometries that closely approximated those of existing objects.[20] Finally, it was shown how it could be used as the basis for a process of automatic design for function, in which an initial starting design was iteratively changed using standard optimization techniques in order to improve its physical properties.[13,14]

This article describes the first stage in a similar program in the area of aerodynamic design. The aim of this article is to illustrate the method using generic geometries that are defined in simple mathematical terms, and which permit the nature of the solution to be illustrated.

The PDE method is a "boundary representation scheme" in that it defines an object by means of description of its surface.[26,27] Usually, this means describing the surface of a complex object in terms of a "patchwork" of simple surface patches which, when using the conventional spline surface methods of computer-aided geometric design,[28–30] often have to be "trimmed" in order to close the surface.[31] The PDE method adopts a boundary-value approach to surface design, thus the starting point in the design process is the definition of a series of space curves that constitute the "character lines" on the surface of the aircraft that mark the boundaries between adjacent surface patches. We will illustrate the way in which the PDE method can be used to define geometry by using the example of a double delta wing configuration typical of a supersonic transport.

The geometry considered in this example is simplified in that only the large-scale features of the aircraft are reproduced. No effort has been made to reproduce small-scale features or features such as detailed wing sections. The aim of this article is to demonstrate the potential of the method.

A. Function Boundary Conditions for PDE Solution

In this illustrative example we consider an aircraft shape made up of three patches: 1) a fuselage, 2) an inner wing, and 3) an outer wing. For simplicity we will use a fuselage that is defined algebraically (discussed later). The inner and outer wings will be generated using the PDE method. The character lines that form the boundaries between adjacent surface patches are 1) the curve where the inner and outer wing meet, 2) the curve where the inner wing meets the fuselage, and 3) a curve at the tip of the outer wing.

The fuselage is generated as a surface of revolution whose axis is parallel to the x axis, and where the y and z coordinates of points on the surface are related by

$$y^2 + z^2 = a^2 \tag{3}$$

with

$$a(\chi) = a_0 \sin[(\pi/18)(17\chi + 1)] + a_1 \sin[(3\pi/18)(17\chi + 1)] \tag{4}$$

where a_0, a_1 are constants and χ is a parameter that lies in the range $0 \leq \chi \leq 1$. Note that as χ varies in the range $1 \rightarrow 0$ we move from the front towards the rear of the aircraft, and the cylindrically symmetric fuselage exhibits the waisted profile characteristic of aircraft designed for supersonic flight.

The curve where the outer wing and inner wing meet we will take to be a plane curve (z = constant), having the shape of a simple airfoil described parametrically, thus,

$$\begin{aligned} x(\theta) &= ch \sin(\theta/2) \\ y(\theta) &= -(t/2)\sin(\theta) + (6.75)(cam)x(\theta)[ch - x(\theta)] \\ &\quad \times [ch - x(\theta)]/ch^3 \\ z &= a_0 + h1 \end{aligned} \tag{5}$$

where the parameter θ varies in the range $0 \leq \theta \leq 2\pi$; cam is a parameter that controls the degree of (cubic) camber of the airfoil; ch is a parameter that sets the chord length of the airfoil; and $h1$ is a constant determining the span of the inner portion of the wing.

The second character line lies on the surface of the fuselage. It is given parametrically by the equations

$$\begin{aligned} x_f(\theta) &= (b/ch)x(\theta) - xd \\ y_f(\theta) &= yd + (3b/2ch)y(\theta) \\ z_f(\theta) &= \sqrt{a^2[\chi(\theta)] - y_f^2(\theta)} \end{aligned} \tag{6}$$

where a is given in terms of χ by Eq. (5) and χ is given in terms of θ by

$$\chi(\theta) = [bx(\theta)/rlch] + (xte/rl) \tag{7}$$

where b, xte, and rl are constants. The second character line is basically a curve upon the fuselage, whose projection onto a vertical plane containing the fuselage's axis is an airfoil of the same type as character line 1, but scaled by a factor (b/ch) and offset with respect to the first character line by the vector ($-\boldsymbol{xd}$, $\boldsymbol{yd}$). The quantity xte determines the position of the fuselage with respect to character lines 1 and 2.

The third character line lies at the tip of the outer wing. It is given parametrically by the equations

$$\begin{aligned} x_t(\theta) &= xt + \left[\frac{xtlx(\theta)}{ch}\right]\cos(a\omega) \\ y_t(\theta) &= -\left[\frac{xtlx(\theta)}{ch}\right]\sin(a\omega) \\ z_t &= a_0 + h1 + h2 \end{aligned} \tag{8}$$

where $h2$ is a constant that determines the span of the outer wing, xt is a constant that determines the fore/aft position of the wingtip, xtl is a constant that controls the length of the wingtip, and $a\omega$ is a constant that determines the amount of "washout." We can see from the previous equations that as θ varies in the range $0 \rightarrow 2\pi$, the wingtip is closed at a straight line of length xtl.

Two wing surfaces are generated between these three character lines: the inner and outer surfaces of the double-delta. The inner wing is generated by solving Eq. (2) using boundary conditions obtained from character lines 1 and 2, and the outer wing is generated by solving Eq. (2) using boundary conditions obtained from the character lines 2 and 3. We solve Eq. (2) over a rectangular region in the (u, v) parameter plane, e.g., $[0, 1] \times [0, 2\pi]$, and choose the boundary conditions so that we map from the boundaries of the rectangular region in parameter space to the boundaries of the corresponding surface patch in physical space.

For simplicity we will assume that on the boundaries of the surface patches

$$\theta(\text{curve parameter}) = v(\text{surface parameter}) \tag{9}$$

so that for the inner wing $\boldsymbol{X}(0, v)$ is given by Eq. (5), and $\boldsymbol{X}(1, v)$ is given by Eqs. (6); similarly for the outer wing. We will consider wings that are closed at the trailing edge, and hence, look for solutions with the property that $\boldsymbol{X}(u, 0) = \boldsymbol{X}(u, 2\pi)$. Thus, we do not require boundary conditions for $\boldsymbol{X}(u, 0)$ and $\boldsymbol{X}(u, 2\pi)$ other than the periodicity condition.

B. Derivative Boundary Conditions for PDE Solution

Equation (2) is fourth-order, hence, we require boundary conditions on the normal derivatives of $\boldsymbol{X}(u, v)$ in the (u, v) parameter plane, which in this case means boundary conditions on $\boldsymbol{X}_u$. The form of these derivative boundary conditions is chosen so that there is continuity of surface normal between adjacent surface patches. Now the surface normal is determined by the vector product $\boldsymbol{X}_u \times \boldsymbol{X}_v$, and in this example $\boldsymbol{X}_v$ is given on the boundary by the conditions on $\boldsymbol{X}$, which means that the surface normal at the edge of a surface patch is determined by the boundary conditions we choose to impose on $\boldsymbol{X}_u$.

On character line 1, which lies at the junction of the outer and inner wings, the derivative boundary conditions are

$$\begin{aligned} x_u(\theta) &= (s1)x(\theta)/2 \\ y_u(\theta) &= 0 \\ z_u(\theta) &= -s1 \end{aligned} \tag{10}$$

On character line 2, which lies on the fuselage, the derivative boundary conditions are

$$\begin{aligned} x_u(\theta) &= (s2v)(y_f)_\theta \\ y_u(\theta) &= -(s2v)(x_f)_\theta \\ z_u(\theta) &= \frac{(a)(a_{xi})x_f - (y_f)_\theta y_f}{(a^2 - y_f^2)^{1/2}} \end{aligned} \tag{11}$$

where $(y_f)_\theta = \partial y_f/\partial\theta$, $a_{xi} = \partial a/\partial(xi)$, and $s2v$ is given by

$$s2v = s2 \sin(\theta/2) \tag{12}$$

$s2$ being another adjustable design parameter.

On character line 3, which lies at the wingtip, the derivative boundary conditions are as follows:

$$\begin{aligned} x_u(\theta) &= (sxt)\left[\frac{x(\theta)xtl}{ch} - \frac{swt}{sxt} - \frac{xtl}{2}\right]\cos(a\omega) \\ y_u(\theta) &= (-sxt)\left[\frac{x(\theta)xtl}{ch} - \frac{swt}{sxt} - \frac{xtl}{2}\right]\sin(a\omega) \\ z_u(\theta) &= 0 \end{aligned} \tag{13}$$

sxt is a parameter that controls the magnitude of the surface derivatives at the tip; increasing it forces the surface away from the tip.

With little effort it may be shown that these equations ensure that there is continuity of surface normal between the outer and inner wing at character line 1, and between the inner wing and the fuselage at character line 2. Furthermore the quantities $s1$, $s2$, sxt, swt, and $a\omega$ are adjustable parameters whose values may be changed in order to control the shape of the generated surfaces, with the continuity of surface normal still being maintained between adjacent patches. The effect of some of these parameters is illustrated in Sec. IV, but we note here that $a\omega$ controls the degree of "washout" and swt controls the extent to which the wingtips exhibit a "swallow-tail" effect.

Thus, to produce the inner wing, Eq. (2) is solved subject to the function and derivative boundary conditions (6) and (11) imposed at the fuselage, and function and derivative boundary conditions (5) and (10) imposed at the outer wing/inner wing junction. And to produce the outer wing, Eq. (2) is solved subject to the function and derivative boundary conditions (5) and (10) imposed at the wing junction, and function and derivative boundary conditions (8) and (13) imposed at the outer wing/inner wing junction.

The control that may be exercised over the shape of the aircraft surface by altering the various "design" parameters in the problem formulation is discussed in Sec. IV.

IV. Results

In Sec. III we defined a generic aircraft shape in terms of 20 parameters. As we shall see in this section the geometry so generated captures many of the basic features of the type of aircraft we are considering, at least at this broad level of resolution.

An initial geometry is shown in Fig. 1, and the corresponding values of the design parameters are shown in Table 1. Notice that it displays the double delta shape characteristic of a supersonic transport. In the next few sections the effect of varying some of the design parameters upon the geometry is shown. For all of the geometry changes that are considered, it should be kept in mind that perfect connectivity (up to continuity of surface normal) is maintained between the patches.

Because limitations of space the changes produced by only a few of the design parameters introduced in Sec. III are considered. However, if the reader cares to implement the method as described earlier, he/she can ascertain the effect of the various design parameters on the aircraft geometry.

A. Variation of Aircraft Planform

In this section we illustrate the effect of varying some of the parameters that control the aircraft's planform. We will do this by changing from the geometry shown in Fig. 1 to the geometry shown in Fig. 2d, through those intermediate geometries shown in Figs. 2a–2c.

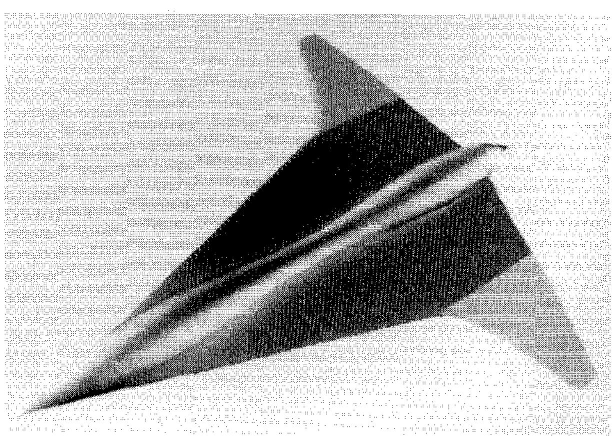


Fig. 1 Initial aircraft geometry.

Table 1 Values of design parameters for aircraft geometry shown in Fig. 1

Parameter	Value
a_0	1.3
ch	6
cam	0
$h1$	4
xte	2
xd	1
xt	1.5
$a\omega$	0
$s1$	2
sxt	2
a_1	0.5
$r1$	25
b	20
$h2$	6
rl	25
yd	0.1
xtl	1
swt	−1
$s2$	2
a	0.3

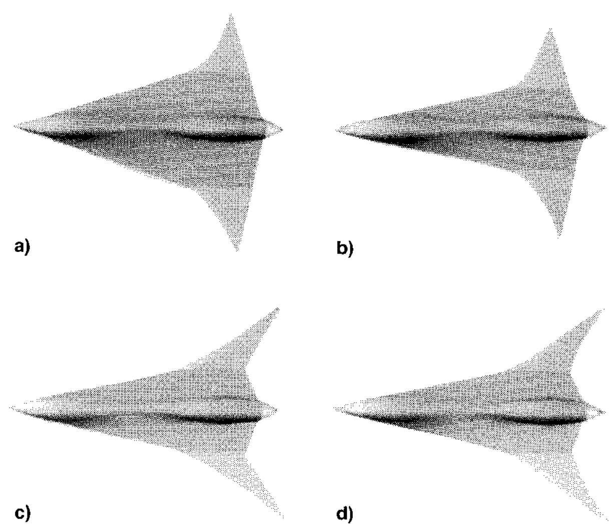


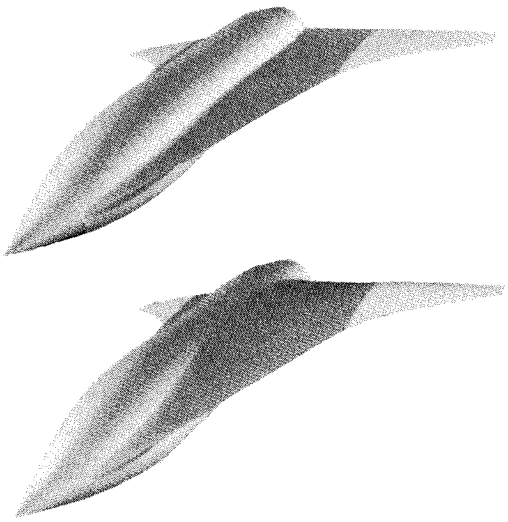
Fig. 2 Variation in aircraft planform.

First, consider reducing the length of the wingtips. This may be effected by changing the value of xtl from 1 to 0.2 (see Fig. 2a). Next, the width of the inner wing is reduced by changing $h1$ from 4 to 2.5, and the wingtip is rounded off by increasing the value of the tip derivative parameter sxt from 2 to 7.6 (see Fig. 2b). Now the wings are swept back by changing the value of xt from 1.5 to −3.5 (see Fig. 2c). Then, finally, we may produce a swallow-tail effect by setting the value of swt to −4.1 (see Fig. 2d).

B. Geometry of Wing/Body Blend

In this section we consider the region where the inner wing blends smoothly to the fuselage. In particular we consider the changes in the geometry of this region that are produced by varying various design parameters, particularly those controlling the wing camber and the derivative boundary conditions imposed at the fuselage. For the sake of example, we will fatten the fuselage (by setting the value of a_0 to 3.1) so that it no longer looks realistic, but so the geometry changes are readily visible.

In Fig. 3 the effect of increasing the camber parameter *cam* from 0 to −0.4 is shown; note that the three surface patches


a)

b)

Fig. 3 Introducing camber to wing: a) no camber and b) cubic camber.

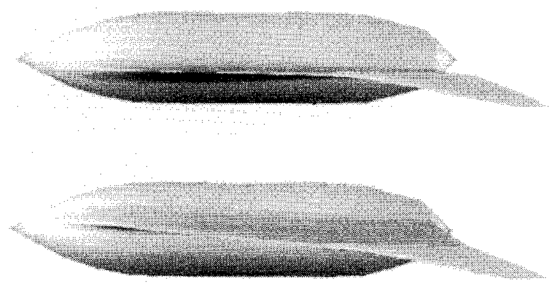


Fig. 4 Changing the wing position.

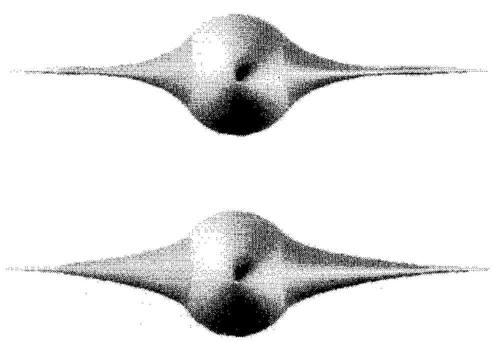


Fig. 5 Changing the surface derivatives at the fuselage.

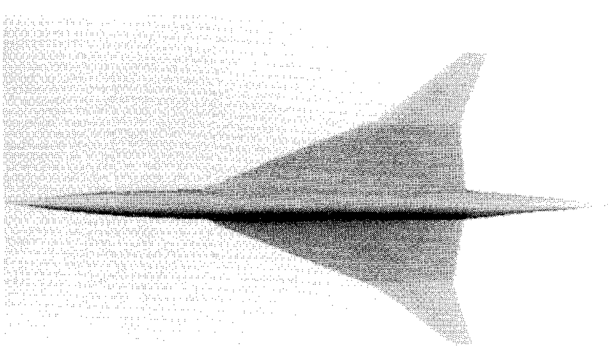


Fig. 6 SST configuration.

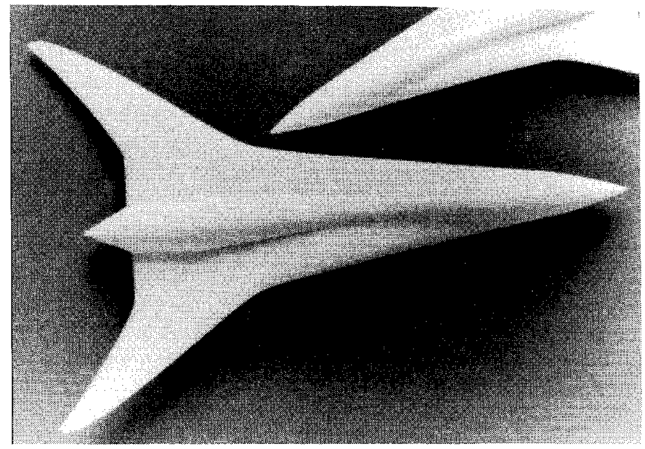


Fig. 7 Polycarbonate model of aircraft geometry.

making up the aircraft continue to blend smoothly together. Figure 4 shows the effect of changing the parameter yd, which controls the vertical position of the wing from -0.645 to 0.727.

Finally, Fig. 5 shows the influence of the slope parameter $s2$. This parameter controls the magnitude of the surface derivatives X_u at the fuselage: increasing $s2$ causes the inner wing to be pushed away from the character line where the wing blends to the fuselage. Note that for the sake of illustration the height of the blend at the fuselage has been exaggerated.

V. Conclusions

It is the aim of this article to indicate the potential of the PDE method as a system for the design of complex free-form shapes. Its virtue is that it can efficiently parametrize such shapes in terms of a small set of design parameters. This is in contrast to conventional methods that typically use polynomial patches,[28–30] and which often require hundreds of individual patches to represent a complex object. Furthermore, such spline-based patches often do not meet exactly at their boundaries, and consequently, have to be trimmed or "stitched" in order to close the surface of the object.[31] The PDE method, however, does not suffer from such problems, since its boundary-value approach means that adjacent surface patches meet exactly at their boundaries. The geometrical design of an object begins with the specification of character lines, and then the surfaces that span the space between them are required by the imposed boundary conditions to meet at their boundaries exactly.

As mentioned in Sec. I, we believe that the techniques described previously are a step towards integrating geometrical with functional design in an automatic process: the step being to parametrize geometry in terms of a small number of parameters, which makes numerical optimization computationally feasible. Elsewhere, case studies, admittedly considering simplified model problems, have demonstrated how this can be achieved for the design of objects for their mechanical[15,16] and hydrodynamical properties.[14] We hope that this work can be extended to aerodynamics.

Of course, when compared to the geometry of a real aircraft, the examples presented in this article are highly simplified. In reality, e.g., in the design of a wing, the variation of its section geometry along its span is usually highly influenced by past experience and current trends in airfoil technology. However, this does not apply so strongly to other parts of the airframe. The wing/body fairing has many of the attributes and requirements of a blend surface: it must form a smooth transition between the wing and the fuselage, while at the same time satisfying requirements on its strength, weight, and aerodynamic properties. Furthermore, previous work has shown that the PDE method can, in fact, approximate existing foil-like geometries to a high degree of accuracy, in particular the geometries of marine propellers.[20,21] It will be the aim of future work to show how well the PDE method can approximate specific features that are deemed crucial in existing geometries, e.g., specific airfoil cross sections.

Although much work is still required to demonstrate that the method is capable of producing the specific geometries found in aircraft, we believe that even now it still has value, in that by using the method it is possible to rapidly produce and numerically test the aerodynamic properties of a wide range of radically different design alternatives. Note that once the initial parametrization was produced, each different aircraft geometry shown in this article was created interactively at a silicon graphics workstation in real-time. As a further indication of the range of geometries that are accessible to the particular parametrization described earlier, consider Fig. 6, which shows a possible configuration for a high speed civil transport.

Finally, one of the virtues of the method is that it can easily produce a triangular-faceted approximation of the surface, from which a physical prototype using the techniques of layered manufacture[32] may be produced, which may then be cast in metal for tunnel testing. Figure 7 shows such a prototype created by the technique of selective laser sintering.

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